Opinion Formation with the Evolution of Network

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Abstract: We analyze users' behavior in Tianya online forum, and find great fluctuations exist in the amount of agents that participate in interactions. Therefore we present a model that includes the co-evolving dynamics of network and opinions. New nodes are added to network gradually, promoting the total size of the system. On the other side, old nodes which feel tired of the topic may withdraw interactions. The rule of continuous opinion exchanges is applied in the dynamics, and only active agents can update their opinions and build connections with new nodes. Simulation results show topological and individual dissipative features of our model are analogous to the Tianya network. Our network is a scale-free and small-world network. Unlike other opinion models, if the tolerance parameter is small, more disordering will be caused in our model with the time elapsed. For a large tolerance parameter, the number of clusters declines, and only one macroscopic-size cluster exists at last. We also realize a first-order phase transition where the second-largest cluster is incorporated into the largest one.

Keywords: Opinion dynamics; network evolution; Deffuant model; dissipative structure

1. Introduction

Nowadays, Internet has greatly developed, and a large part of people search, read and publish information on it instead of traditional media. Many applications on Internet break forth and get improved. At present, Web 2.0 has become very popular, including microblogs[1], bulletin board systems (BBS)[2], social networking sites[3], etc. Lots of researchers concentrate on these real networks, aiming to investigate the topological characteristics and network growth. It is proved that Internet is not only a scale-free network, but also displays small-world properties[4]. Barabasi and Albert (BA) presented a network model with preferential attachment mechanism. The degree of nodes follows a power law, and the clustering coefficient declines swiftly as the network size increases[5]. Watts and Strogatz built a small-world network by randomly reconnecting edges of regular lattices, but this network doesn't have a power-law degree distribution[6]. Inspired by their work, plenty of models were proposed, describing a certain kind of real network[7,8].

At the same time, information interactions in these networks also attract much attention, such as rumor diffusion, opinion evolution. Opinion dynamics tries to explain and forecast the formation of public opinion, exploring the condition of phase transition[9]. Statistical physics is applied to attain macroscopic features of a finite-size system[10]. There are already abundant basic opinion models with different interacting rules, that is, the Sznajd model[11,12], the voter model[13,14], the Deffuant model[15], the Hegselmann-Krause model[16], and so on. Based on these dynamic models, extensive research has been carried out. In [17], mobility of agents is introduced to the CODA model in small-world networks, so agents are allowed to change their positions. The number of extremists in the group decreases significantly. R. Lambiotte presented a non-conservative voter model where the opinion of an agent depends non-linearly on the fraction of disagreeing neighbors, making the average magnetization between zero and consensus[18]. Wu et al. supposed that agents would adopt the most common opinion that exceeds a threshold value, and he found an intermediate metastable state during the evolution[19]. Sven Banisch used empirical data to explain opinion exchange dynamics, and his model performed similar transient opinion configurations as the electoral performance of candidates[20].

Interplays between opinion evolution and topology evolution have been taken into consideration. The reconnecting of network may accelerate the formation of large clusters, or divide these clusters into fragment[21,22]. Present studies consider the system is fixed with a finite size. Although agents can change their ideas and relations, but the number of agents in interactions keeps identical all along. However, in real networks, the underlying network is growing during the opinion dynamics. For instance, on Internet new users enter the network step by step, but meantime old users may lose their interests, and drop out of discussions, preventing the occurrence of consensus. Users can only remain active in a finite period of time, and the size of population changes all the time.

Fluctuations of system size have a vital influence on opinion formation, so the growing of network can not be ignored. In this paper, we put forward an opinion model in consideration of the growth and recession of network. We research into the problems of network characteristics and opinion clusters, and we also analyze the dissipative features of Tianya BBS, and compare this real network with our model.

We will introduce our work as follows. We analyze the features of Tianya BBS in Section 2. Section 3 presents a model with the coevolution of network and opinions. In Section 4 simulation results about the model are included. We close the paper in Section 4 with concluding remarks.

2. Interacting network of Tianya BBS

Tianya BBS (www.tianya.cn) has become one of the most famous online forums in China. As a typical representative of Web2.0, Tianya BBS provides a public circumstance for users to present their opinions and discuss with other people. Till August 2011, there are more than 56 million users existing in this network, and the number of online active users stays above one million. Because of this large quantity of population, Tianya network is sometimes the original source of online emergency. Therefore analyzing the characteristics of this network can help us to understand the formation of public opinion on Internet. We collected data from one section (Zatan) of this network by our directed crawler. Theme posts and their replies were gained, as well as user relations. More than three million posts and replies from 2002 to 2009 were downloaded after 8 hours' crawling. In the network, nodes represent users, and the relation between nodes is built after a user replies a post author.

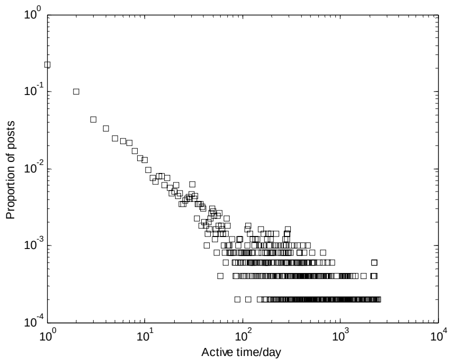

Figure 1. Distribution function of number of post's active days for Tianya BBS

As mentioned in [23], the network has a power-law degree distribution, and high clustering coefficients. It is scale-free and small-world. Here we focus on the evolution of network and the variation of active users. We pay attention to the size of interacting users. Figure 1 shows the distribution of posts' active time in Tianya network. The active time of a post begins when the author creates it, and the post dies out after the last user replies it. It is obvious that the number of posts' active days decays as a power law. A large part of posts become extinct in less than 3 months, but several posts can last for more than 1000 days. After 3 years, there are still some users discussing in the post. Therefore the interacting time of topics differs a lot from each other.

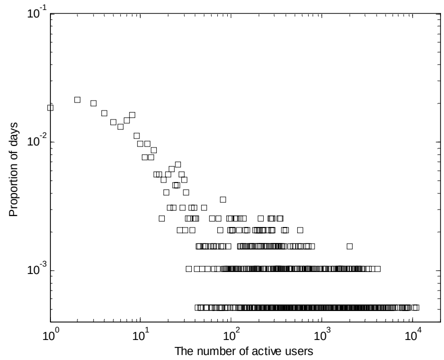

Figure 2. Distribution function of number of daily active users for Tianya BBS

We calculate the amount of daily active users that at least publish a post or a reply on that day, as shown in Figure 2. The distribution of active users on every day follows a power law. The distribution function has a longer tail, implying the network has a relatively large size of interacting population. The longe tail results from the phenomenon that many new users register and login the forum. There are still 0.002 proportion of days on which more than 700 users interact with other people in the section of Tianya BBS. We notice that the largest number of daily active users exceeds 10 thousand. However, it is very frequent that only 2 or 3 users discuss some topics in the network on a day. Figure 3 illustrates the distribution of amount of users' active days on which they join in discussions. Clearly, this amount declines as a power law. Most of users drop out of the evolution of network quickly. Users will never be active in the network for more than 1000 days. Thus users have extremely imbalanced active time.

From above results, we realize that the interacting system is no longer fixed, and its size changes every day. A great many new users arrive day by day, and plenty of existing users lose their interests gradually and withdraw the network. Only a few users can persist in interacting for long. Opinion formation is accompanied by the growth and recession of network, and actions and reactions exist in the process of opinion and network evolution. Since many users keep away from discussions, their opinions can not be changed, and they don't have enough opportunities to persuade others. The dissipation of network prevents the formation of consensus, and may lead the system to fragmentation. This agrees with the situation that consensus is not always reached on Internet.

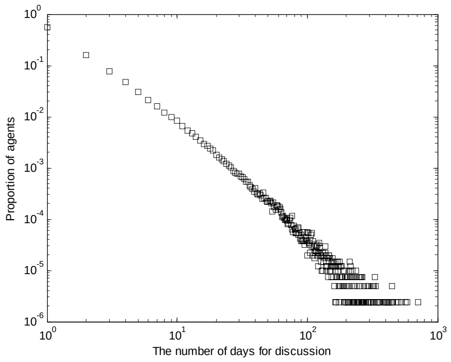

Figure 3. Distribution function of number of users' days for discussion in Tianya BBS

3. The model

The real society especially Internet is a nonlinear dissipative system. Agents meet new people every day, and contact with different communicatees. Thus their local neighborhoods are changing during the whole evolution. The underlying network mediates opinion exchanges, but it is influenced by individual opinions notably. The dynamics of network need to be included in the process of opinion interaction.

In the Deffuant model, a population of N agents are located on nodes of a network that describes the relations between agents. Individual opinions are continuous taking value from $(0,1)$. An agent's opinion σ expresses its attitude towards an event in society. The opinion $\sigma > 0.5$ means the agent consents the event, and vice versa. In an update, an agent i and one of its neighbors j are selected at random. If the distance d between the two agents' opinions stays below the parameter ε of bounded confidence, that is, $d = \left|\sigma_j - \sigma_i\right| \le \varepsilon$, these two agents will change their opinions in accordance with each other. Otherwise, no opinion exchange takes place. Time is increased by 1 after N such updates.

Based on the Deffuant interacting rule, we introduce our model as follow. The evolution begins in an initially small-size network. At first, there are several agents in the network, and they are randomly connected. Since agents may lose their activity and drop out of discussion, only active agents can take part in opinion exchanges or network evolution. In an update, an active agent i with its node degree k and one of its neighbors j (no matter what activity the neighbor is) are picked out. If they have similar opinions, agent i will adapt its opinion close to its neighbor. The active agent changes its opinion following the rule, i.e. $\sigma_i = \sigma_i + 0.5 \cdot \left(\sigma_j - \sigma_i\right)$, if $d \le \varepsilon$. On the contrary, when the difference of their opinions exceeds the tolerance value, the active agent i may become inactive and stop discussing. People always would like to communicate with others of similar belief, but they avoid to debate with contrarians[21,24]. Agent i quits the discussion with the probability $1/k$ that is inversely proportional to the degree of agent. It is natural that positive agents contact with others frequently, and make connections with lots of friends. Therefore they have a large degree, and are reluctant to stop interactions. Meanwhile, at each time step, a new agent is added to the network, randomly connected to n active nodes that already exist in the network.

As mentioned above, the dynamics of network and opinions take place synchronously. If no active agent exists in the network, new nodes can not connect to old nodes, so the dynamics is frozen immediately. Otherwise, new nodes are introduced until the total system size N reaches a given value. However, the dynamics will go on unless all agents become inactive or no opinion exchange occurs, implying the system achieves a stable state.

4. Simulation Results

Now we will implement Monte-Carlo simulations to investigate the elaborate coevolution of agents' opinions and their relations. All individual opinions are assigned randomly from $(0,1)$ in the beginning. Without loss of generality we assure that there are 20 active agents in the initial network, and this quantity will not influence the macroscopic dynamics. We will explore the network characteristics and opinion distribution of our model.

4.1. The Network

We study the effect of opinion evolution on the network, and how individual bounded confidence changes connections between agents. Though these inactive agents participate in interactions no longer, their opinions are already published, and can influence other active agents. Therefore the statistical features of these dormant agents should be taken into account.

Figure 4 represents the final degree distribution $P(k)$ of the network created by our model. $P(k)$ of this network is power-law, that is, $P(k) \propto k^{-\gamma}$. The power exponent γ is 2.721, and it depends on individual bounded confidence. Though new nodes are linked to old ones at random, but all nodes will become inert with a rate that is closely related to their degrees. It means nodes having more neighbors are less likely to keep away from the evolution. As only active nodes can receive links, nodes with a large degree have more priority to enlarge their connectivity. The action can be regarded as a modified mechanism of preferential attachment. Moreover, the shortest average path length of our network is 3.48, and the clustering coefficient is 0.2660 that stays much higher than the BA network. The power-law degree distribution and high clustering suggest our network is a scale-free and small-world network.

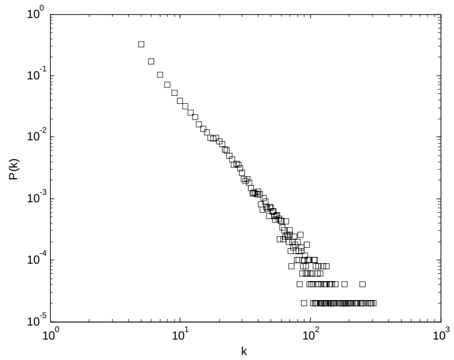

Figure 4. Degree distribution of our network, $n = 5$, and $\varepsilon = 0.2$. The final size of all agents N is 1000. The result is an average of 100 realizations.

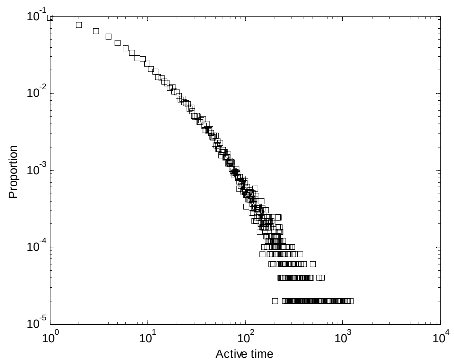

Figure 5. Distribution of number of nodes' time steps for discussion in our model, $N = 1000$, $n = 5$ and $\varepsilon = 0.2$.

The proportion of agents with different active time also decays as a power law (Fig. 5). Large parts of agents only attend the topic for a while, and not more than one agent on average can persist in discussing for longer than 100 time steps. Not all agents which enter the network early have a long active time, so individual activity has nothing to do with its existence time. This outcome coincides with that of Tianya network.

4.2. Opinions

With the evolution of network, agents holding similar opinions merge together to form large clusters, but large divergence between individual opinions may make active nodes stop discussing. As shown in Fig. 6, for a large tolerance value, the number of opinion clusters increases gently at the early stage. New nodes enter the network gradually with random opinions. At that time, though existing active agents change their opinions and form large clusters, the process of network evolution plays a significantly main role to create new opinion clusters. After time 800, the process of opinion evolution is in a dominant position. In the stable state, several large clusters are left in the system. The number of opinion clusters will reach a plateau after a transitory decreasing. However, with small ε, the amount of opinion clusters always rises till a stable level, leading to the fragmentation state. The system becomes more chaotic with the coevolution of network and opinions. This means if a dynamic system is lack of trust, more disordering will be caused with time elapsed. Gini coefficient that is used in economics first, can be regarded as the distribution characteristic of agents holding different opinions[15]. The larger the Gini coefficient is, the fewer opinions agents focus on. The Gini coefficient increases linearly with time, and at last it will level off (in Fig. 7). Unlike the amount of opinion clusters, the process of opinion evolution takes an important effect on Gini coefficient all the time.

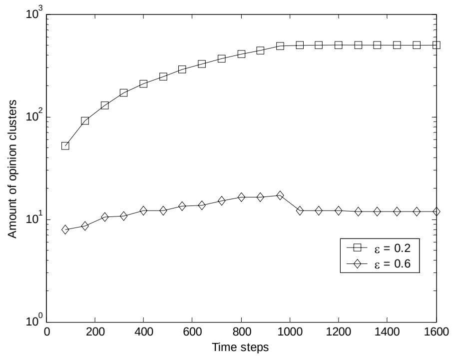

Figure 6. Time plot of number of opinion clusters, $N = 1000$ and $n = 10$.

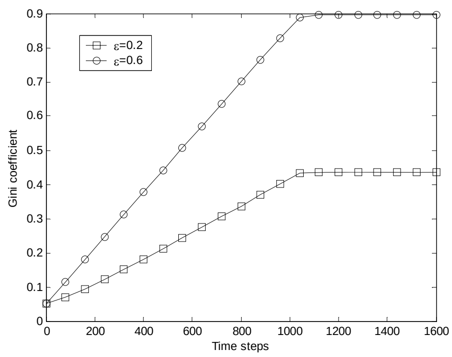

Figure 7. Gini coefficient as a function of tim, $N = 1000$ and $n = 10$.

From Fig. 8, the size of largest opinion cluster increases with the tolerance parameter monotonously. The size of second-largest cluster has a small increment with ε, but this cluster will be incorporated into the largest one. When $\varepsilon > 0.5$, the system approaches the consensus state with one single macroscopic-size cluster present in the final state. The second-largest cluster becomes most apparent for $\varepsilon = 0.2$, and two large-size clusters coexist in the end. Raising average degree of the network enlarges macroscopic clusters, but can not influence the first-order phase transition in the thermodynamic limit.

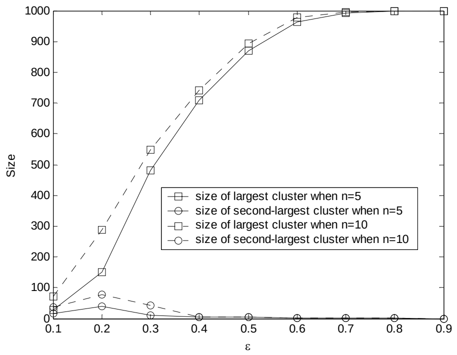

Figure 8. Size of largest opinion cluster and second-largest opinion cluster versus ε , for $N = 1000$.

5. Conclusions

In this paper, we have studied the effect of network evolution on opinion dynamics, and the reaction of opinions to network. Growth and recession exist in the dynamic process, making the system size change each time. Only active agents are likely to change their opinions. We have explored the topological features of the network and dissipative behavior of agents, and opinion distribution and convergence are also shown.

Clearly, the network in our model has a power-law degree distribution, and topological and users' dissipative features are in agreement with some real BBS networks. The Gini coefficient increases with time, implying individual opinions are changed close to each other. However, the number of opinion clusters increases with the occurrence of new agents, so the network growing takes the main effect at that stage. A large tolerance parameter can decrease the number of clusters, enlarging macroscopic-size clusters. In future work, we will use statistical natural language processing to analyze the real BBS topic data and verify our model.

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